Circuits and Boolean Expressions

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Lesson Focus

Boolean logic is essential to understanding computer architecture. It is also useful in program construction and Artificial Intelligence. This lesson is a gentle introduction to formal logic using Boolean notation, and Circuits. Students learn the basic rules by playing the role of logic gates in a half adder and full adder. Free logic gate construction software available online can be incorporated optionally.

Age Levels

Recommended for ages 8 – 10
Also appropriate for ages 11 – 13

Objectives

Introduce students to:

- Simple Boolean expressions, AND, OR, NOT.
- Adders and Full Adders.
- Binary Arithmetic.

Anticipated Learner Outcomes

Students will be able to:

- Construct simple circuits on paper, and use simulators.
- Develop a circuit diagram from a Boolean expression.
- Develop a truth table for a circuit diagram.
- Demonstrate how an adder and full adder work.

Alignment to Curriculum Frameworks

See attached curriculum alignment sheet.

Internet Connections

- http://www.dailymotion.com/video/x2ssz7l (Binary arithmetic)
- http://academo.org/demos/logic-gate-simulator/ (Suggested logic gate simulator)
- https://www.youtube.com/watch?v=qdFmSlFojIw
- http://www.circuitstoday.com/boolean-logic
- http://www.circuitstoday.com/half-adder-and-full-adder

Recommended Reading

- https://en.wikibooks.org/wiki/High_School_Mathematics_Extensions/Logic

Optional Writing Activity

- Which symbol system do you like best: Boolean expressions, Circuits or Truth Tables? Explain.

Circuits and Boolean Expressions

For Teachers:
Teacher Resources

◆ Lesson Objectives

Introduce students to:

- Simple Boolean expressions, AND, OR, NOT.
- Adders and Full Adders.
- Binary Arithmetic.

◆ Materials

- Paper and pencils.
- Index cards sufficient for each student to have 6 of one color, 4 of a second color, and 4 of a third color (plus extras of each color). If index cards are not available, cut heavy stock paper into appropriate sized pieces (construction paper will do.)
- Optional: Computers or mobile devices for using logic simulators.
- Straws, or thin strips used to connect logic gates.

◆ Procedure

The purpose of this lesson is to expose your students to simple logic gates. If you are comfortable with the material, keep in mind that this is a short introduction, not a full-blown discrete mathematics or logic course. Even if you love proofs, please stay away from them unless you are confident your students are motivated. Instructors at the university level often struggle to provide an intuitive understanding of the essential elements: AND, OR, NOT. Students tend to memorize the truth tables and then mechanically do proofs using a 'cross your fingers and hope' strategy. This lesson is intended to help students internalize the basic concepts regardless of the formalism used.

In the first session, students will gain practice in recognizing the rules of simple gates by using a free online simulation, and playing competitive games that work up to the introduction of a half adder and full adder. Students are led to realize that a simple, logical expression produces the same result as the rules of binary arithmetic on single digits. Please have the students create the card deck themselves, because creating the cards through the simple act of drawing the gates, the Boolean expressions, and the truth tables, reinforces these relationships between these concepts.

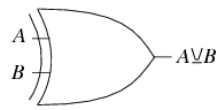
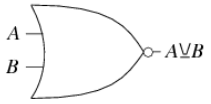
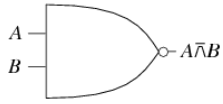
In the second session, students are introduced to solving the logic for a full adder. The culminating activity is a whole class addition of two bit binary numbers, ideally with three bits. You will have to preplan how to divide up your class into two teams. Each team needs a relay of three or four full-adders. Each full adder should consist of two people who problem-solve which outputs to post, based on the inputs they received. Consequently, there should be a minimum of six students per side. Adjust the logistics accordingly by having larger teams choose who will compete for them, or by adding another full-adder to the relay.

Surprisingly, Boolean expressions as logic circuits is not well represented among online instructional videos appropriate for middle school. However, there is a multitude of games, simulations, and physical kits that introduce logic gates. Although this lesson can be done entirely with pencil, papers, and the attached templates for a set of playing

cards, allowing students to explore simple gates through computer-based interaction, enhances their experience.

Session 1

1. Introduce your students to simple binary arithmetic of single digits. Either demonstrate in class, or use this video: http://www.dailymotion.com/video/x2ssz7l. Worksheet 1 provides a guided exercise. Watch the video on the Internet Connections on converting between binary and decimal. If appropriate, include the optional video in Recommended Reading.
2. Distribute Worksheet 1, 6 index cards of one color (the logic cards), 4 index cards of a second color (input cards), and 4 cards of a third color (output cards). Have your students create a set of Logic Cards for themselves, then using the logic cards, follow the instructions on Worksheet 1 to create the 'Gate' cards. The three basic operations NOT, OR, AND can be referenced from the student resource sheet. In pairs or groups of at most four, have them use a Logic Gate simulation such as http://academo.org/demos/logic-gate-simulator/ to create the other three: XOR, NOR, NAND. If the software or a physical kit is unavailable, post the rules on the board as follows. Truth tables can be found on line. Each student should also create two input cards labeled '0' and two input cards labeled '1', two output cards labeled '0', and two output cards labeled '1'. Have students help each other fill out the cards and *make sure each student puts their name on their cards.*

XOR: exclusive OR	True when exactly one input is true, not true otherwise.	
NOR	NOT OR	
NAND	NOT AND	

3. As students complete their cards, assign them to groups of three, but preferably four. Have them collect all of their logic cards into a pile and the input cards into a pile. Review the game rules described on the Worksheet 2.
4. If there's time, as a final activity show them the Boolean expressions for a half adder, and ask them to construct logic gates and truth tables using ANY combination of gates. You choose whether individuals or groups win. The Boolean expressions are:
   S = A XOR B
   C = A AND B
5. Have the groups of student collect their cards and bundle them for next time. Invite students to add cards to the decks and distribute blank index cards as needed.

Session 2

1. Watch the video listed in the Internet Connections on converting binary numbers to decimals. Have students form the same groups as last time and redistribute their card decks. As a warm-up, have them create the half adder as circuits and truth table, again using the cards and straws.
2. Before further play within the group, show the entire class that the truth table looks exactly like the rules for adding two binary digits, and in fact 'S' stands for 'sum' and 'C' stands for 'carry'. Encourage them to play the game with two outputs.
3. Distribute Worksheet 3. It provides the logic gate layout for a full adder and a blank truth table. Ask them to write the Boolean expressions and fill in the truth table. Remind members of the group to help each other.
4. The worksheet encourages them to practice having two group members behave as a full adder. They will receive three inputs and have to produce two outputs, passing on the 'C_{out}'. Let each group determine how they will respond to inputs. (It is ok if they borrow ideas from other groups.)
5. Keeping the groups on the same teams, create two relay teams of three 'full adder pairs'. Follow the directions on Worksheet 3 to play the adder game.
6. Conclude the lesson with a full class discussion about what they learned.

◆ Time Needed

- ✦ 2 sessions, at most 1 hour each. This can be done in a single two-hour session if your students are typically cooperative during group activities.

Student Resource:
Boolean Algebra, and Logic Gates

◆ Binary Arithmetic

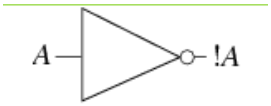
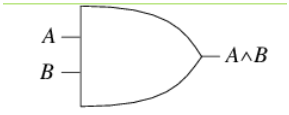
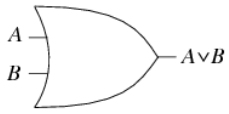
We count in Base 10, with digits 0,1,2,3,4,5,6,7,8,9.
Computers count in Base 2, with digits 0,1. This means they do a lot of addition with carry, but the rules are simpler: 1 + 1 = 10, since we only have two digits we have to carry to the next column. Here are the conversions from between binary and decimal

Binary	Decimal
0	**0**
0	**1**
10	**2**
11	3
100	4
101	5
110	6
111	7
1000	8

Yes, it keeps going.

◆ Boolean Operators and Logic Gates

Computers count in binary because they are made up of logic gates. Very complicated computation can happen with the rule that "TRUE" is the same as "1", and "FALSE" is the same as '0'. There are three simple rules called 'operators' Boolean logic and circuit gates. Each rule takes one or two binary digits (or truth values) and produces a resulting truth-value or digit. Some people like to think in Boolean, others find it easier to think about logic gates, still others like truth tables. As you do this lesson, think about which one makes the most sense to you.

Operator	Rule	Boolean	Logic Gate	Truth Table
NOT	If input is TRUE, output is FALSE	$\overline{A}$ or !A		A | ! A 0 | 1 1 | 0
AND	When all the inputs are TRUE, the output is TRUE, otherwise the output is FALSE. T when all inputs are T, F otherwise	A ∧ B or AB		A | B | A+B 0 | 0 | 0 0 | 1 | 0 1 | 0 | 0 1 | 1 | 1
OR	When all the inputs are FALSE, output is FALSE, otherwise output is TRUE T when either input is T, F otherwise	A ∨ B or A + B		A | B | AB 0 | 0 | 0 0 | 1 | 1 1 | 0 | 1 1 | 1 | 1

Student Worksheet 1:
Binary Arithmetic and Logic Gates

◆ Binary Arithmetic

The student resource page gives you the rules for binary arithmetic. Perhaps you watched a video or your teacher showed you how to add four bit numbers. In your group, challenge each other by making up your own problems such as this one. Convert each number to decimal (using the chart on the student resource page). When the subscript is '2' the number is binary, when the subscript is missing or '10', the number is decimal.

$$01011_2 = 1000_2 + 10_2 + 1_2 = 8 + 2 + 1 = 11_{10}$$
$$01101_2 = 1000_2 + 100_2 + 1_2 = 8 + 4 + 1 = 13_{10}$$
$$11000_2 = 10000_2 + 1000_2 + 1_2 = 16 + 8 = 24_{10}$$

Student Worksheet 2:
The BLT Game (Boolean Logic Truth)

◆ Materials

Six index cards to create Logic Cards. Their color is _____.
Four index cards to create Input Cards. Their color is _____.
Four index cards to create Output Cards. Their color is _____.
Straws to create connections between gates.

◆ Setup

Create Logic Cards for NOT, AND, OR, XOR, NAND, NOR

- Using your student resource page as a guide, create cards for NOT, AND, OR.
- Draw the logic gate on the front; write the name (e.g. NOT), the Boolean expression, and the truth table on the back.
- Using resources provided by your teacher (or perhaps a computer simulation) create cards for XOR, NAND, NOR.
- WRITE YOUR NAME ON THE BACK OF EACH CARD.

Create Input and Output Cards

- Write a large "1" on two of the input cards and two of the output cards.
- Write a large "0" on the remaining input and output cards.
- Collect everyone's logic and input cards. Keep your output cards. Have a paper and pencil handy to keep score, and straws ready to connect logic gates.

◆ Game Play

The player with the most points wins. Players take turns being dealer. The person to the left of the dealer is the problem solver. The remaining players are the challengers.

Dealer: Lay out a circuit by drawing four cards from the deck. Use at least two cards to build a combinational logic diagram by connecting the output of one card with the inputs(s) of another. Write down the Boolean expression for your diagram. Depending on the number of inputs to your left-most logic gate, pick two input cards and place them at the inputs to that gate. You can have only one output.

Problem Solver: Work through the circuit and place either '0' or '1' at the output.

Challenger: Agree or disagree with the output, agree or disagree with the Boolean expression. Use truth tables to prove your assertion if others contest it.

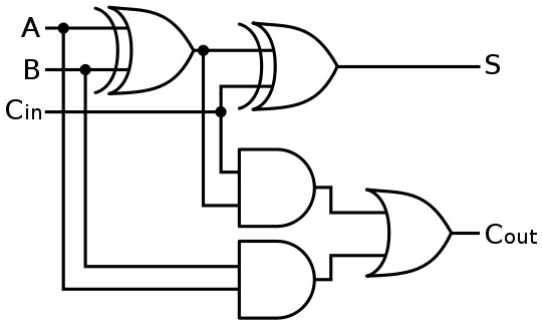
◆ Scoring

- Dealer: 1 point if the Problem Solver got it right, 3 points for the correct Boolean expression.
- Problem Solver: 6 points for the correct answer.
- Challenger(s): 2 points for a successful challenge or for agreeing to the right answer.
- Anyone: 10 points for successfully arguing that the others are wrong.

Student Worksheet 3:
A Full Adder

This is a full adder. It has three inputs and two outputs. Write the Boolean expressions and fill in the truth table.

S = _____ C_{out} _____



A	B	C_{in}		S	C_{out}
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Your class will play full-adder relay. To do that you have to be quick about knowing what the outputs are, based on the inputs you receive. As a team, invent the fastest way to know the outputs given inputs A, B and $C_{in.}$ Hint: think binary arithmetic rather than logic!

Use this space to figure out the Boolean expressions for S and $C_{out.}$

Circuits and Boolean Expressions

For Teachers: Alignment to Curriculum Frameworks

Note: All lesson plans in this series are aligned to the Computer Science Teachers Association K-12 Computer Science Standards, and if applicable also the U.S. Common Core State Standards for Mathematics, the U.S. National Council of Teachers of Mathematics' Principles and Standards for School Mathematics, the International Technology Education Association's Standards for Technological Literacy, the U.S. National Science Education Standards and the U.S. Next Generation Science Standards.

◆National Science Education Standards Grades K-4 (ages 4-9)

CONTENT STANDARD E: Science and Technology

As a result of activities, all students should develop

- Abilities of technological design
- Understanding about science and technology

◆National Science Education Standards Grades 5-8 (ages 10-14)

CONTENT STANDARD E: Science and Technology

As a result of activities, all students should develop

- Abilities of technological design
- Understandings about science and technology

◆Next Generation Science Standards & Practices Grades 3-5 (ages 8-11)

Practice 2: Generating and Using Models

- Use a model to test cause and effect relationships or interactions concerning the functioning of a natural or designed system.

Practice 6: Constructing Explanations and Designing Solutions

- Use evidence (e.g., measurements, observations, patterns) to construct or support an explanation or design a solution to a problem

Practice 7: Engaging in Argument from Evidence

- Construct and/or support an argument with evidence, data, and/or a model.

◆Next Generation Science Standards & Practices Grades 6-8 (ages 11-14)

Practice 2: Generating and Using Models

- Develop and/or use a model to generate data to test ideas about phenomena in natural or designed systems, including those representing inputs and outputs, and those at unobservable scales.

Practice 5: Using Mathematics and Computational Thinking

- Use mathematical representations to describe and/or support scientific conclusions and design solutions.

Practice 6: Constructing Explanations and Designing Solutions

- Construct an explanation using models or representations

Practice 7: Engaging in Argument from Evidence

- Construct, use, and/or present an oral and written argument supported by empirical evidence and scientific reasoning to support or refute an explanation or a model for a phenomenon or a solution to a problem

Circuits and Boolean Expressions

For Teachers: Alignment to Curriculum Frameworks

◆ Principles and Standards for School Mathematics Grades 3-5 (ages 8 - 11)

Algebra Standards

- understand patterns, relations and functions
  - ✦ represent and analyze patterns and functions, using words, tables, and graphs.

◆ Principles and Standards for School Mathematics Gr. 6-8 (ages 11 - 14)

Algebra Standards

- understand patterns, relations and functions
  - ✦ represent and analyze and generalize a variety of patterns with tables, graphs, words, and, when possible symbolic rules

◆ Principles and Standards for School Mathematics (all ages)

Problem Solving Standards

- ✦ Solve problems that arise in mathematics and other contexts

Communication Standards

- ✦ Communicate their mathematical thinking coherently and clearly to peers, teachers and others

◆Common Core State Practices & Standards for School Mathematics (all ages)

- ✦ CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.
- ✦ CCSS.MATH.PRACTICE.MP4 Model with mathematics.

◆ Standards for Technological Literacy - all ages

Nature of Technology

- ✦ Standard 2: Students will develop an understanding of the core concepts of technology

Design

- ✦ Standard 10: Students will develop an understanding of the role of troubleshooting, research and development, invention and innovation, and experimentation in problem solving.

The Designed World

- ✦ Standard 17: Students will develop an understanding of and be able to select and use information and communication technologies

◆CSTA K-12 Computer Science Standards Grades 3-6 (ages 8-11)

5.1 Level 1: Computer Science and Me (L1)

- ✦ Computational Thinking (CT)
  4. Describe how a simulation can be used to solve a problem.
- ✦ Collaboration (CL)
  3. Identify ways that teamwork and collaboration can support problem solving and innovation.

Circuits and Boolean Expressions

For Teachers: Alignment to Curriculum Frameworks

◆CSTA K-12 Computer Science Standards Grades 6-9 (ages 11-14)

5. 2 Level 2: Computer Science and Community (L2)

- Computational Thinking (CT)
  13. Understand the notion of hierarchy and abstraction in computing including high- level languages, translation, instruction set, and logic circuits.
- Collaboration (CL)
  3. Collaborate with peers, experts, and others using collaborative practices such as pair programming, working in project teams, and participating in group active learning activities.