Choose Your Best Way

Lesson Focus

Lesson focuses on how mathematic models help to solve real problems and are realized in computers. Students work in teams to build a graph model of their city map while learning how mathematic models work. Student should be encouraged to use this model to solve real problems.

Lesson Synopsis

The "Choose Your Best Way" lesson explores how to build a mathematic model that helps solve real problems and how to realize algorithmic thinking in computers. Students work in teams to build a graph model of their city map. Students then try to solve a real problem based on the model, evaluate their solutions, and present their reflections to the class.

Age Levels

12-18.

Objectives

- ✦ Learn about discrete mathematic modeling.
- ✦ Learn about graph theory.
- ✦ Learn about computer algorithms.
- ✦ Learn about teamwork and working in groups.

Anticipated Learner Outcomes

As a result of this lesson, students should develop an understanding of:

- ✦ mathematic models
- ✦ graph theory
- ✦ computer algorithmic thinking
- ✦ teamwork

Lesson Activities

Students learn how to solve a real problem in computer with the help of discrete mathematic models. They work in teams to build a graph model of their city map while learning how mathematic models work.

Resources/Materials

- ✦ Teacher Resource Documents (attached)
- ✦ Student Resource Sheets (attached)
- ✦ Student Worksheets (attached)

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Internet Connections

✦ TryComputing (www.trycomputing.org)
✦ Graph theory tutorials of the University of Tennessee at Martin
  (http://www.utm.edu/departments/math/graph)

Recommended Reading

✦ A Beginner's Guide to Discrete Mathematics (ISBN: 978-0817642692)
✦ Introductory Graph Theory (ISBN: 978-0486247755)
✦ Beginning Programming All-In-One Desk Reference For Dummies (ISBN: 978-0470108543)
✦ Schaum's Outline of Graph Theory (ISBN: 978-0070054899)

Optional Writing Activity

✦ Explore how the Internet finds the website you want when you tell your browser to go to CNN.com or Google.com. Write an essay or paragraph explaining how this relates to the graph problem described in this unit.

Credits

✦ This lesson plan was developed by Shuang Liu, an IEEE Student Member from Hannover Germany (Region 8) as part of the IEEE TryComputing.org Lesson Plan Competition.

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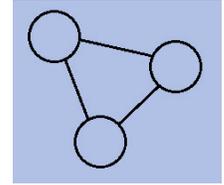
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◆ Lesson Goal

The "Choose Your Best Way" lesson explores how to build a mathematic model that helps solve real problems and how to realize an algorithmic thinking in computers. Students work in teams to build a graph model of their city map. Once they completed teams should try to solve real problems based on the model, evaluate their solutions, and present their reflections to the class.

◆ Lesson Objectives

- ✦ Learn about discrete mathematic modeling.
- ✦ Learn about graph theory.
- ✦ Learn about computer algorithms.
- ✦ Learn about teamwork and working in groups.

◆ Materials

- ✦ Student Resource Sheets
- ✦ Student Worksheets
- ✦ One set of materials for each group of students:
  - o A city map.
  - o Ruler, eraser, pencil, pencil sharpener, marker pen.

◆ Procedure

1. Show students the Student Reference Sheets. These may be read in class, or provided as reading material for the prior night's homework.
2. Divide students into groups of four or five.
3. To introduce the lesson, discuss with students the use of networks in daily life. For instance road and rail systems, connections via social media and computer networks.
4. Teams will consider their worksheet.
5. Student teams complete an evaluation/reflection sheet and share their experiences with the class.
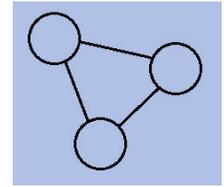
◆ Time Needed

- ✦ Two 45 minute sessions.

◆ Advanced Option

The teacher makes a presentation using electronic map software or automotive navigation system to give students a deep impression.

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For Teachers:
Sample to the students worksheets

◆ **A Sample Graph Corresponding to a Map**
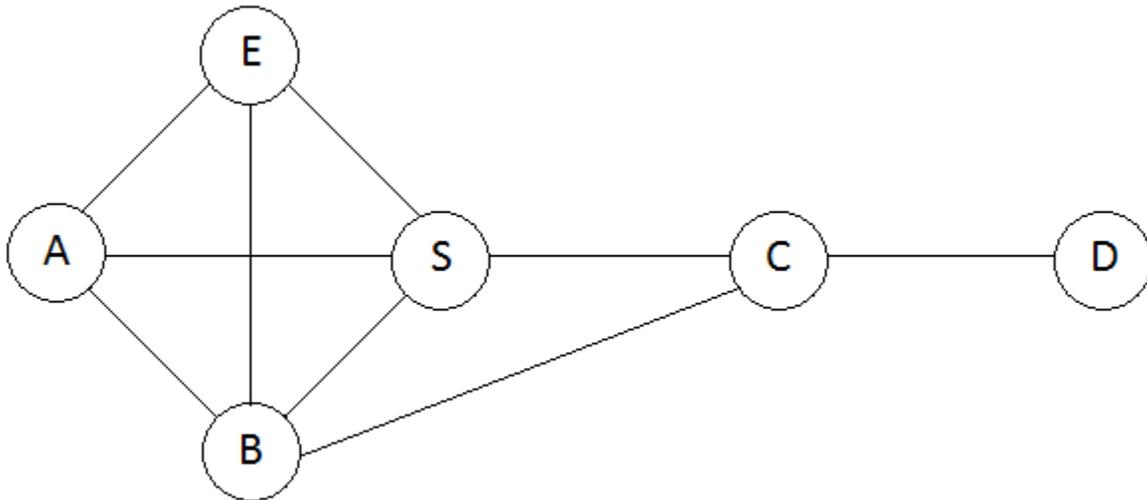
◆ **The Distance Sheet**

Unit: m

distance	school	A's home	B's home	C's home	D's home	E's home
school	-	200	400	100	-	600
A's home	200	-	300	-	-	500
B's home	400	300	-	400	-	700
C's home	100	-	400	-	200	-
D's home	-	-	-	200	-	-
E's home	600	500	700	-	-	-

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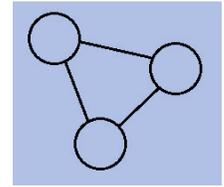
For Teachers:
Sample to the students worksheets

◆ **The Results**

First method:
Order: School-C-D-C-B-A-E-School, Total Distance: 2300 meters.
Second method:
Order: School-C-D-C-School-A-E-A-B-A-School, Total Distance: 2600 meters.

Attention: This is the so-called Traveling Salesman Problem. Both of the routes cannot ensure the most optimal solution.

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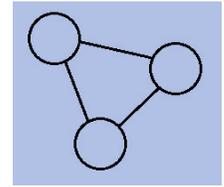
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◆ Discrete Mathematics

Discrete mathematics is the study of mathematical structures that can be counted with whole numbers, like your fingers and toes. Such as integers, graphs and statements in logic are the objects studied in discrete mathematics. Discrete mathematics is useful in studying and describing objects and problems in computer sciences.

◆ Graph Theory

Graph theory is the study of graphs. A graph refers to a collection of vertices and a collection of edges that connect pairs of vertices. The paper written by Leonhard Euler on the Seven Bridges of Königsberg and published in 1736 is regarded as the first paper in the history of graph theory.

◆ The Seven Bridges of Königsberg

During the eighteenth century, the city of Königsberg in East Prussia (now Kaliningrad, Russia) was divided into four sections by the river Pregel. Seven bridges connected these regions. The challenge was to find a walk through the city that would cross each bridge once and only once. The islands could not be reached by any route other than the bridges. Euler proved that the problem has no solution.
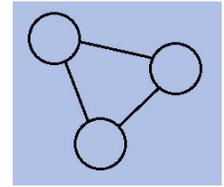
◆ Eulerian Path and Eulerian Circuit

In graph theory, an Eulerian path is a path in a graph which visits every edge exactly at once. Similarly, an Eulerian circuit is an Eulerian path which starts and ends on the same vertex.
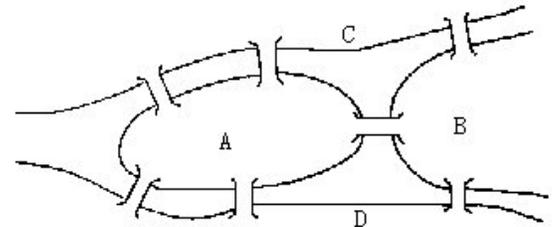
◆ Hamiltonian Path and Hamiltonian Circuit

In graph theory, a Hamiltonian path is a path in a graph which visits each vertex exactly at once. A Hamiltonian circuit is a cycle which visits each vertex exactly once and returns to the starting vertex.

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◆ **Graph Layout**

Read the city map carefully.

1. Use a marker or pen to circle your school and home of each member in your team in the map. Find the shortest roads between each pair of the places. (If the number of your team is less than four, please choose some typical places instead, such as parks, libraries, etc.)

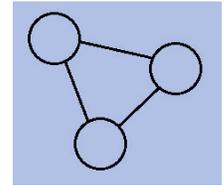2. Sketch the corresponding graph below.
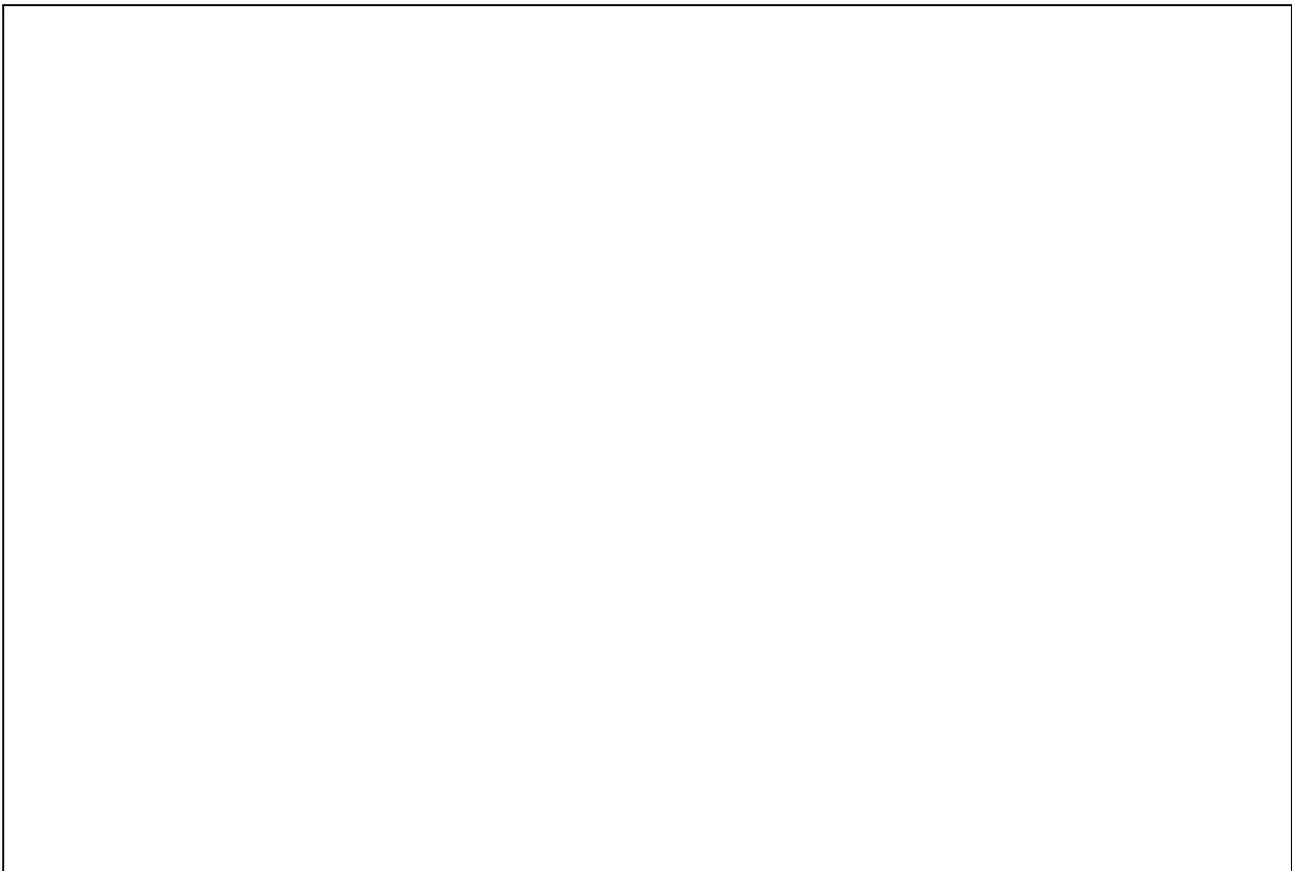
3. Fill in the blanks with the words "places" and "roads".
       The edges of this graph represent _____, while the vertexes represent _____.

4. Read the student resource sheets. Then discuss whether your graph contains an Eulerian Path or an Eulerian Circuit.

What about Hamiltonian Path and Hamiltonian Circuit?

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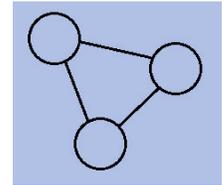www.tryengineering.org

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5. In graph theory, if every pair of distinct vertices is connected by a unique edge in a graph, then the graph is called a complete graph.
Is your graph a complete graph, or does your graph contain such a complete graph?

6. Measure the distances between each pair of the places and fill in the data on the following sheet. Please pay attention to the scale of the map. If there is no direct way between any two places, write down "-" instead.

Unit: _____

distance	school	_____'s home	_____'s home	_____'s home	_____'s home	_____'s home
school	-					
_____'s home		-				
_____'s home			-			
_____'s home				-		
_____'s home					-	
_____'s home						-

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7. Now, do you think you can remake the graph just using the data in this sheet? If your answer is yes, try to do it. Then you may understand how a graph can be stored in computers. This is one possible solution, of course there are also many other solutions.

◆ Search the Shortest Path

Assume that now you are at school with your friends and you forget to bring your homework. You need to go back home to pick it up and return to school. Since all of you are very good friends, you prefer to be together, that means you need to start from school to everyone's home and finally return to school.

8. Think about how to find the shortest path that covers all of the places and write down your idea. Then try to find the shortest path.

9. Search the path using the following two different methods and calculate the length of the path.

First method:
Start the path from school and mark the school as visited;
The path always goes to the next place that does not be marked as visited and is the nearest neighboring place until all the places are marked as visited;
If there is no further place available, return to the last place;
Then path end at the school.

The order of places: _____
Length of the path: _____

Second method:
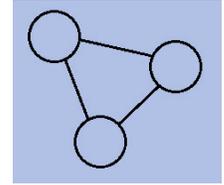Find the shortest distance between any two places and mark the places as visited;
Repeat to find another shortest distance between any two places (at least one of the places should not be marked), until all the places are marked as visited;
Check if any two places can be connected by the paths. If not, find the shortest way to connect them.

The order of places: _____
Length of the path: _____

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◆ **Review**

The paper on the Seven Bridges of Königsberg published in 1736 by Leonhard Euler is regarded as the cornerstone of graph theory. After hundreds of years of development, graph theory is now the fundamental of computer science and is widely used in applications. Many algorithms have been found for problems in different fields, such as electrical networks, coding theory, operations research and computer programming.

◆ **Evaluation**

Complete the evaluation questions below:

1. Compare the results of both methods. Which result is better?

2. Compare your answer with other groups. Are the answers the same?

3. If the results are not equal to the best solution you have obtained in task 8, which result do you prefer? And which method do you prefer?

4. Assume that you have a map of thousands places. Which method do you think is more efficient?

5. Do you think the methods are appropriate for computers? Why?

6. Which applications do you think your work is significant for?

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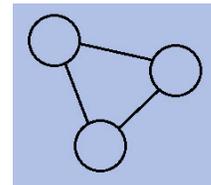
Teacher Resource:
Alignment to Curriculum Frameworks

Note: Lesson plans in this series are aligned to one or more of the following sets of standards:
- U.S. Science Education Standards (http://www.nap.edu/catalog.php?record_id=4962)
- U.S. Next Generation Science Standards (http://www.nextgenscience.org/)
- International Technology Education Association's Standards for Technological Literacy (http://www.iteea.org/TAA/PDFs/xstnd.pdf)
- U.S. National Council of Teachers of Mathematics' Principles and Standards for School Mathematics (http://www.nctm.org/standards/content.aspx?id=16909)
- U.S. Common Core State Standards for Mathematics (http://www.corestandards.org/Math)
- Computer Science Teachers Association K-12 Computer Science Standards (http://csta.acm.org/Curriculum/sub/K12Standards.html)

◆ **Principles and Standards for School Mathematics**
   As a result of activities, all students should develop
   Number and Operations Standard
   ✦ Compute fluently and make reasonable estimates
   Geometry Standard
   ✦ Specify locations and describe spatial relationships using coordinate geometry and other representational systems
   ✦ Use visualization, spatial reasoning, and geometric modeling to solve problems
   Measurement Standard
   ✦ Understand measurable attributes of objects and the units, systems, and processes of measurement
   ✦ Apply appropriate techniques, tools, and formulas to determine measurements.
   Problem Solving Standard
   ✦ Apply and adapt a variety of appropriate strategies to solve problems.
   ✦ Solve problems that arise in mathematics and in other contexts.
   Representation
   ✦ Use representations to model and interpret physical, social and mathematical phenomena

◆ **Common Core State Standards for School Mathematics Grades 2-8 (ages 7-14)**
   Measurement and data
   - Measure and estimate lengths in standard units.
   ✦ CCSS.Math.Content.2.MD.A.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
   ✦ CCSS.Math.Content.2.MD.A.3 Estimate lengths using units of inches, feet, centimeters, and meters.

◆ **Common Core State Standards for Mathematics Grades 9-12 (ages 14-18)**
   Number and Quantity
   - Quantities
   o Reason quantitatively and use units to solve problems
   ✦ CCSS.Math.Content.HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

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◆ **Standards for Technological Literacy – All Ages**

The Nature of Technology

✦ Standard 2: Students will develop an understanding of the core concepts of technology.

✦ Standard 3: Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study.

The Designed World

✦ Standard 17: Students will develop an understanding of and be able to select and use information and communication technologies.

◆**CSTA K-12 Computer Science Standards Grades 6-9 (ages 11-15)**

5. 2 Level 2: Computer Science and Community (L2)

✦ Computational Thinking: (CT)

3. Define an algorithm as a sequence of instructions that can be processed by a computer.

7. Represent data in a variety of ways including text, sounds, pictures, and numbers.

8. Use visual representations of problem states, structures, and data (e.g., graphs, charts, network diagrams, flowcharts).

✦ Collaboration (CL)

3. Collaborate with peers, experts, and others using collaborative practices such as pair programming, working in project teams, and participating in group active learning activities.

4. Exhibit dispositions necessary for collaboration: providing useful feedback, integrating feedback, understanding and accepting multiple perspectives, socialization.

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