"Boolean Algebra is Elementary"

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Lesson Focus

Sherlock Holmes delighted in saying 'It's elementary, my dear Watson'. This lesson provides a brief overview of how Boolean algebra provides the basis for artificial intelligence reasoning. The rules of propositional logic are introduced in the context of the kind of 'AI' found in role-playing games both on the computer and off.

Age Levels

Recommended for 14 – 18
Also appropriate for upper Middle School (e.g. age 13)

Objectives

Introduce students to:

- The rudiments of Boolean Algebra.
- Both a formal and intuitive understanding of implication (p -> q).
- Mechanical (computational) approaches to the standard rules of inference, informal resolution theorem proving.

Anticipated Learner Outcomes

Students will be able to:

- Describe a scenario as facts and propositions.
- Use Boolean algebra to represent a world state.
- Determine whether a proposition is true or false using standard rules of inference.

Alignment to Curriculum Frameworks

See attached curriculum alignment sheet.

Internet Connections

Propositional Logic from Carneades.org.

- https://youtu.be/b1YZ83zABk8?list=PLz0n_SjOttTcjHsuebLrl0fjab5fdToui - Basics
- https://youtu.be/mL6lyrF5-w8?list=PLz0n_SjOttTcjHsuebLrl0fjab5fdToui -Truth Tbls
- https://youtu.be/SjLtbpEq_WE?list=PLz0n_SjOttTcjHsuebLrl0fjab5fdToui - Negation
- https://youtu.be/YveDZxMKuAs?list=PLz0n_SjOttTcjHsuebLrl0fjab5fdToui - Disjunction
- https://youtu.be/NyU_C73BHBc?list=PLz0n_SjOttTcjHsuebLrl0fjab5fdToui- Conjunction
- https://youtu.be/do5vRAnntKI?list=PLz0n_SjOttTcjHsuebLrl0fjab5fdToui- Implication

Recommended Reading

- http://www.cs.odu.edu/~toida/nerzic/content/web_course.html
- http://www.cs.mun.ca/~kol/courses/2742-f08/studysheet-t1.pdf

Optional Writing Activity

- This activity introduces Boolean Algebra as a formalism for reasoning with proposition. Make a case for or against: is this a sufficient way to represent the real world (or even a virtual one), and the reason for your choice.

Boolean Algebra is Elementary

For Teachers:
Teacher Resources

◆ Lesson Objectives

Introduce students to:

- The rudiments of Boolean algebra.
- Both a formal and intuitive understanding of implication (p -> q).
- Mechanical (computational) approaches to the standard rules of inference.

◆ Materials

- Access to the Internet to watch the videos listed under Internet Connections.
- Paper, pencils.
- Index cards, sufficient for groups of 3 or 4 students to have 30 cards that can be shuffled per group. Heavy grade paper that can be cut into playing card sized pieces will suffice.
- One six-sided die per group. (Alternatively use a coin, heads is true, tails is false).

◆ Procedure

The purpose of this lesson is to give students a flavor of how Boolean algebra can be used to represent a rule base in a virtual world (a video game or simulation). The goal is to motivate them to appreciate how logic governs the virtual worlds they take for granted, not to teach them to do proofs or to master the basic principles of formal logic. Let them play amongst themselves - be their coach and guide, rather than an instructor. That said, before the first session, you are encouraged to create your own virtual world from the instructions on Worksheet 1, and find volunteers to play the game described on Worksheet 2.

In Session 1, students become familiar with propositions and the laws of Boolean algebra. In Session 2, they practice applying the laws in order to answer questions about the state of the world they have created.

Session 1:

1. Depending on the background of your students, have them watch some or all of the videos listed under Internet Connections. Perhaps assign viewing for homework beforehand, or allow them to individually decide which ones to watch.
2. Divide the students into groups of three or four. Explain that they will be building virtual worlds. As a whole class, brainstorm some themes for these worlds. An absurd one based on Alice in Wonderland appears in the resources. Do not simply give them this (incomplete) world to play. The learning hinges on having them create their own virtual world. Have each student sign up to develop one of these themes - allow at most five students per theme. Break larger groups who want to work on the same theme, into teams of three to five. Pairs and singles will not work.
3. Distribute Worksheet 1 and index cards. Spend your time coaching the groups.

Session 2:

1. Distribute Worksheet 2 and have the students play their own games. Allow them to modify the propositions to improve the game. If time permits, have groups play each other's games.
2. Have a wrap-up discussion about whether Boolean Algebra is sufficient for representing a world, and ask them what more is needed?

Sample Cards for Teachers

The following incomplete set of cards is included to illustrate examples of propositions and implications in a world. They should be used as model to create a teacher sample set of at least 20 cards to develop the skills to guide students in creating theirs. Please do not use or extend this set and assign your students to play it. The power of the learning occurs in *creating* the set, not in playing the game, especially in a world that might not be of interest. Also please bear in mind that if you have D&D or even Pokémon aficionados in the class they will pick this up quickly. This sample is intentionally small to fit on the page. Try to make the cards at least 2X3 inches. (8 X 5 cm.)

A The cat is green	B The bucket is empty
C You are next to the well	D The food is poisoned
E I am in a magical land	F The cat is thirsty
A + !E The cat is green and I am not in a magical land	AE The cat is green and I am in a virtual world
A -> E If the cat is green then I am in a magical land If the cat is green implies I am in a magical land	AD -> F The cat is green and the food is poisoned -> The cat is thirsty
C -> !B Your are next to the well -> The bucket is not empty	!B-> !F The bucket is not empty -> The cat is not thirsty

Time Needed

- 2 sessions, at most 1 hour each. This can be done in a single two-hour session if your students are typically cooperative during group activities.

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Student Resource:

Boolean Operator:

Operator	Rule	Boolean	Truth Table
NOT Negation	If input is TRUE, output is FALSE	¬A !A	A | ¬A F | 1 T | F
AND Conjunction	When all the inputs are TRUE, output is TRUE, otherwise output is FALSE. T when all inputs are T, F otherwise	A ∧ B or AB	A | B | A+B F | F | F F | T | F T | F | F T | T | T
OR Disjunction	When all the inputs are FALSE, output is FALSE, otherwise output is TRUE T when either input is T, F otherwise	A ∨ B or A + B	A | B | AB F | F | F F | T | T T | F | T T | T | T
A IMPLIES B	When A is false, or WHEN B is TRUE, output is TRUE otherwise FALSE. The implication is true when A is false or B is true	A → B ¬A ∨ B !A +B	A | B | A → B F | F | T F | T | T T | F | F T | T | T

Laws of Boolean Algebra:

Commutative	*A + B = B + A*	*A B = B A*
Associate	*(A + B) + C = A + (B + C)*	*(A B) C = A (B C)*
Distributive	*A (B + C) = A B + A C*	*A + (B C) = (A + B) (A + C)*
Identity	*A + A = A*	*A A = A*
	AB + A!B = A	*A(A+ B) +(A+!B) = A*
Redundance	*A + A B = A*	*A (A + B) = A*
	A +! A B = A + B	*A(!A + B)= A* B
De Morgan's	*!(A +B) = ! A !B*	*!(A B) = ! A + !B*

Note the notation for NOT (! Or ¬)

Student Worksheet 1:
Propositions and Boolean Rules

Video games and simulations keep track of the world state through propositions that are true or false. The rules of the game are written as implications. The 'AI' in a game (the artificial intelligence) manipulates a rule base to determine what is happening. Boolean Algebra is a formalism used to reason about a virtual world. This worksheet guides your team as you construct a small virtual world using Boolean algebra. The rules to allow others to play your game are on Worksheet 2.

◆ Propositions

Any statement of fact is a proposition. "The cat is green" is a simple fact or proposition that is probably not true in the real world, but could be true in a virtual world. Another proposition is "I am in a magical land". Your team will have to construct a set of propositions that can be either true or false in your world. Variables are used as shorthand. For example:

'A' stands for "The cat is green", 'B' stands for "I am in a magical land".

◆ Negation, Conjunction, Disjunction

These are fancy words for 'not', 'and', 'or'. These words have very precise meanings.

"The cat is not green" can be written as !A.
"The cat is green or I am not in a magical land" can be written as A + !B.
"The cat is green and I am in a virtual world" can be written as AB.

◆ Implications

These are tricky beasts. They do not represent causality, although it is tempting to think so. Implications can be true or false. For example:

"If the cat is green then I am in a magical land." This is an implication. You can also say "The cat is green implies I am in a magical land." Either can be written as A->B.

The implication is true when the cat is not green or when I am in a magical land.

Assignment 1: Work with your team to fully understand the rules of logic. Don't just rely on or memorize the truth tables. Start by writing propositions and discuss how sentences that include negations, conjunctions, and disjunctions would be expressed.

Assignment 2: Create a virtual world by creating a list of propositions. Start with simple statements like "The cat is green". Think about the circumstances under which your simple statements are true or false to create more complex propositions. For example:

The cat is green and the bucket is empty (A and C)
The cat is green and the bucket is not empty (A and !C)

Construct between 10 and 25 propositions that describe your world. Make sure to include simple statements and implications. Write each proposition on an index card. Write (or type) all of the rules on a single page as well.

Student Worksheet 2:
It's an Elementary Game

Sherlock Holmes liked to say "It is elementary my dear Watson." Sargent Friday on Dragnet liked to say "Just the facts ma'am." In both cases the detectives wanted to separate out the propositions that were true from the ones that were false.

In a virtual world, the AI engine decides your fate based on two situations 1) which propositions are true, because it knows them to be true, and 2) which propositions are true or false based on the implications in the rule base. For example, remember the green cat? Can you deduce the color of the cat based on whether you are in a magical place? If the correct implication is in the list of propositions (the rule base) then you can.

- In this session you will begin to test whether your rule base is consistent: that is, whether you can deduce simple statements without contradictions. In game design, testers explore every possible combination of values for the simple statements. This is one big truth table. Rather than fill out the truth table, you will play a game.

◆ **Game Play: 3 or 4 people**

The player with the most points wins. Players take turns being dealer. The person to the left of the dealer is the problem solver. The remaining players are the challenger.

Dealer: Shuffle the index cards, and pick a card from the deck. This is the question card. The problem solver has to answer the question whether the proposition is true, false, or can't be determined. Pick three more cards from the deck. For each of these cards, roll the die. If the number is even the proposition is false, otherwise the proposition is true. Put the true propositions in one column, and the false ones in a second column. Make a third column out of all the remaining propositions. These are the 'unknowns'.

Problem Solver: Put the question card at the top of the unknown pile. Based on the propositions that are true or false, try to move the question card to the 'true' or 'false' pile. To help you do this, move any cards you can from the 'unknown' pile to either the 'true' or 'false' pile. The dealer can help you do this.

Challenger: Watches whether the move is legal. Use truth tables to prove your assertion if others contest it.

◆ **Scoring:**

- Dealer: 1 point if the Problem Solver got it right, 3 points for the correct Boolean expression.
- Problem Solver: 6 points for the correct answer
- Challenger(s): 2 points for a successful challenge or for agreeing to the right answer.
- Anyone: 10 points for successfully arguing that the others are wrong.

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For Teachers:

Alignment to Curriculum Frameworks

Note: All lesson plans in this series are aligned to the Computer Science Teachers Association K-12 Computer Science Standards, and if applicable also the U.S. Common Core State Standards for Mathematics, the U.S. National Council of Teachers of Mathematics' Principles and Standards for School Mathematics, the International Technology Education Association's Standards for Technological Literacy, the U.S. National Science Education Standards and the U.S. Next Generation Science Standards.

◆National Science Education Standards Grades 5-8 (ages 10 - 14)

CONTENT STANDARD E: Science and Technology

As a result of their activities, all students should develop

- Understandings about science and technology

◆National Science Education Standards Grades 9-12 (ages 14-18)

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As a result of their activities, all students should develop

- Understandings about science and technology

◆Next Generation Science Standards & Practices Grades 6-8 (ages 11-14)

Practice 2: Generating and Using Models

- Develop and/or use a model to generate data to test ideas about phenomena in natural or designed systems, including those representing inputs and outputs, and those at unobservable scales.

Practice 5: Using Mathematics and Computational Thinking

- Use mathematical representations to describe and/or support scientific conclusions and design solutions.

Practice 6: Constructing Explanations and Designing Solutions

- Construct an explanation using models or representations

Practice 7: Engaging in Argument from Evidence

- Construct, use, and/or present an oral and written argument supported by empirical evidence and scientific reasoning to support or refute an explanation or a model for a phenomenon or a solution to a problem

◆Next Generation Science Standards & Practices Gr.9-12 (ages 14-18)

Practice 2: Generating and Using Models

- Develop, revise, and/or use a model based on evidence to illustrate and/or predict the relationships between systems or between components of a system.

Practice 6: Constructing Explanations and Designing Solutions

- Apply scientific ideas, principles, and/or evidence to provide an explanation of phenomena and solve design problems, taking into account possible unanticipated effects.

Practice 7: Engaging in Argument from Evidence

- Construct, use, and/or present an oral and written argument or counter-arguments based on data and evidence.

◆ Principles and Standards for School Mathematics Gr. 6-8 (ages 11 - 14)

Algebra Standards

- understand patterns, relations and functions

- represent and analyze and generalize a variety of patterns with tables, graphs, words, and, when possible symbolic rules

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For Teachers:
Alignment to Curriculum Frameworks

◆ Principles and Standards for School Mathematics Grades 9-12 (ages 14 - 18)

Algebra Standards

-represent and analyze mathematical situations and structures using algebraic symbols

- write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil in simple cases and using technology in all cases.
- use mathematical models to represent and understand mathematical relationships.
- use symbolic algebra to represent and explain mathematical relationships

◆ Principles and Standards for School Mathematics (all ages)

Problem Solving Standards

- Solve problems that arise in mathematics and other contexts

Communication Standards

- Communicate their mathematical thinking coherently and clearly to peers, teachers and others

◆Common Core State Practices & Standards for School Mathematics (all ages)

- CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.
- CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.
- CCSS.MATH.PRACTICE.MP4 Model with mathematics.

◆ Standards for Technological Literacy - all ages

Nature of Technology

- Standard 2: Students will develop an understanding of the core concepts of technology

The Designed World

- Standard 17: Students will develop an understanding of and be able to select and use information and communication technologies

◆CSTA K-12 Computer Science Standards Grades 6-9 (ages 11-14)

5. 2 Level 2: Computer Science and Community (L2)

- Computational Thinking (CT)

  13. Understand the notion of hierarchy and abstraction in computing including high- level languages, translation, instruction set, and logic circuits.

  14. Examine connections between elements of mathematics and computer science including binary numbers, logic, sets and functions.

For Teachers: Alignment to Curriculum Frameworks

- Collaboration (CL)
  3. Collaborate with peers, experts, and others using collaborative practices such as pair programming, working in project teams, and participating in group active learning activities.

◆CSTA K-12 Computer Science Standards Grades 9-12 (ages 14-18)

5.3 Level 3: Applying Concepts and Creating Real-World Solutions (L3)

5.3.A *Computer Science in the Modern World (MW)*

- Computing Practice and Programming (CPP)
  12. Describe how mathematical and statistical functions, sets, and logic are used in computation.

5.3.B *Computer Science Concepts and Practices (CP)*

- Computers and Communications Devices (CD)

  2. Identify and describe hardware (e.g., physical layers, logic gates, chips, components).